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## Laplace of Integration of f(t)

$$\mathcal{L}\int f(t)\,dt = \frac{F(s)}{s} + \frac{\int f(t)\,dt}{s}\Big|_{t=0}$$

Proof-

$$\mathcal{L}\int f(t)\,dt = \int_0^{\infty}\left[\int f(t)\,dt\right] e^{-st}\,dt$$

$$= \left[\left[\int f(t)\,dt\right]\cdot\frac{e^{-st}}{-s}\right]_0^{\infty} - \int_0^{\infty} f(t)\cdot\frac{e^{-st}}{-s}\,dt$$

$$F(s) = \frac{1}{s}\int f(t)\Big|_{t=0} + \frac{1}{s}\int_0^{\infty} f(t)\,e^{-st}\,dt$$

$$= \frac{\int f(t)\big|_{t=0}}{s} + \frac{1}{s}F(s)$$

$$= \frac{F(s)}{s} + \frac{\int f(t)\big|_{t=0}}{s}$$

This is known as real integration theorem.

## → Laplace of Unit Step

$$\mathcal{L}\,u(t) = \int_0^{\infty} u(t)\,e^{-st}\,dt$$

$$= \frac{e^{-st}}{-s}\Big|_0^{\infty}$$

$$F(s) = \frac{1}{s}$$

If $f(t) = K$

then $F(s) = \frac{K}{s}$

## → Laplace of Impulse

– Impulse is signal of infinite magnitude with zero duration $t \to 0$

Also $\delta(t) = \frac{d}{dt}\cdot u(t)$

$$\mathcal{L}\,\delta(t) = \int_0^{\infty}\delta(t)\,e^{-st}\,dt$$

$$= 1.$$

Also $\mathcal{L}\,\delta^{n}(t) = s^{n}$

where $\delta^{n}(t) =$ $n^{th}$ derivative of $\delta(t)$

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$$\cos wt = \frac{e^{jwt} + e^{-jwt}}{2}$$

## Laplace of $\cos wt$

$$\mathcal{L} \cos wt = \frac{1}{2}\int_0^\infty \left(e^{jwt} + e^{-jwt}\right) e^{-st}\, dt$$

$$= \frac{1}{2}\left[\int_0^\infty e^{jwt-st}\, dt + \int_0^\infty e^{-jwt-st}\, dt\right]$$

$$= \frac{1}{2}\left[\int_0^\infty e^{-(s-jw)t}\, dt + \int_0^\infty e^{-(s+jw)t}\, dt\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-jw} + \frac{1}{s+jw}\right]$$

$$\frac{1}{2}\left[\frac{s+jw - s+jw}{s^2+w^2}\right] = \frac{\cancel{2}}{\cancel{2}}\,\frac{jw}{s^2+w^2}$$

$$= \frac{1}{2}\left[\frac{s+jw}{(s-jw)(s+jw)} + \frac{s-jw}{(s+jw)(s-jw)}\right]$$

$$= \frac{1}{2}\left[\frac{s+\cancel{jw}+s-\cancel{jw}}{s^2+w^2}\right] = \frac{\cancel{2}s}{\cancel{2}(s^2+w^2)}$$

$$= \frac{s}{s^2+w^2}$$

$$\mathcal{L} \sin wt = \int_0^\infty \frac{e^{jwt} - e^{-jwt}}{2j} \cdot e^{-st}\, dt$$

$$= \frac{1}{2}\left[\int_0^\infty e^{jwt-st}\, dt - \int_0^\infty e^{-jwt-st}\, dt\right]$$

$$= \frac{1}{2j}\left[\frac{1}{s-jw} - \frac{1}{s+jw}\right]$$

$$= \frac{\cancel{2j}\, w}{\cancel{2j}\,(s^2+w^2)}$$

→ Laplace of Hyperbola

$$\mathcal{L} \cosh at = \int_0^\infty \frac{e^{at} + e^{-at}}{2}\, e^{-st}$$

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$$\mathcal{L}\,\sinh at = \int_0^\infty \sinh at\, e^{-st}\, dt$$

$$= \int_0^\infty \frac{e^{+at} - e^{-at}}{2}\, e^{-st}\, dt$$

$$= \frac{1}{2}\left[\int_0^\infty e^{-(s-a)t}\, dt - \int_0^\infty e^{-(s+a)t}\, dt\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right]$$

$$= \frac{1}{2}\, \frac{s+a-s+a}{s^2-a^2}$$

$$\boxed{\mathcal{L}\,\sinh at = \frac{a}{s^2-a^2}}$$