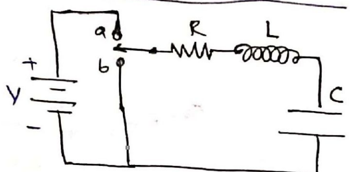
Time Domain Solution of RLC Circuit

When the s/w is at 'a' the circuit has gained steady state



$\therefore\ V_c(0^-) = V = V_c(0^+)$ and then capacitor acts as open so

$$i(0^-) = 0 = i(0^+)$$

When s/w is moved to position 'b'

Applying KVL

$$R\,i(t) + L\cdot\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt = 0 \quad \text{——(I)}$$

differentiating

$$R\cdot\frac{d}{dt}i(t) + L\cdot\frac{d^2 i(t)}{dt^2} + \frac{1}{C}i(t) = 0$$

or

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\cdot\frac{di}{dt} + \frac{i}{LC} = 0$$

The characteristic eqn.

$$\left(D^2 + \frac{R}{L}\cdot D + \frac{1}{LC}\right)i = 0$$

or

$$D^2 + \frac{R}{L}D + \frac{1}{LC} = 0$$

$$D_1, D_2 = \frac{-R/L \pm \sqrt{(R/L)^2 - 4/LC}}{2}$$

Therefore

$$i(t) = K_1 e^{D_1 t} + K_2 e^{D_2 t} \quad \text{——(II)}$$

K_1 and K_2 are arbitrary constants determined by initial conditions

$$\boxed{i(0^-) = i(0^+) = 0, \quad V_c(0^-) = V_c(0^+) = 0}$$

Put these in eqn. (I)

$$R\,i(0) + \frac{1}{L}\frac{d}{dt}i(0) + \frac{1}{C}\int i(t)\Big|_{t=0} = 0$$

$$0 + \frac{1}{L}\frac{di(0^+)}{dt} + V = 0$$

$$\boxed{\therefore\ \frac{di}{dt}(0^+) = -VL \text{ Amp/sec}}$$

Applying initial condition in (II) at $t=0$

$$0 = K_1 + K_2 \quad \text{——(III)}$$

differentiating eqn. (II)

$$\frac{di}{dt} = K_1 D_1 e^{D_1 t} + K_2 D_2 e^{D_2 t}$$

at $t=0$ $\quad \frac{di}{dt} = -VL$

Substituting

$$-VL = K_1 D_1 + K_2 D_2 \quad \text{——(IV)}$$

from (III) and (IV) the value of K_1 and K_2 can be determined and substituted in (II).

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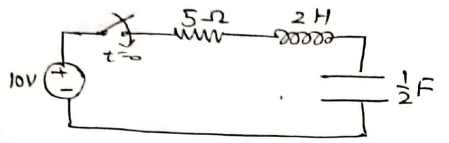
Q. Find the expression for current $i(t)$ for $t > 0$ if s/w is closed at time $t = 0$



Applying KVL

$$\therefore Ri + L\cdot\frac{di}{dt} + \frac{1}{C}\int i\,dt = 10$$

$$5i + 2\cdot\frac{di}{dt} + 2\int i\,dt = 10$$

differentiating we get

$$\frac{d^2i}{dt^2} + 2.5\,\frac{di}{dt} + 2i = 0 \quad \text{—(I)}$$

write the characteristic Eqn.

$$D^2 + 2.5D + 1 = 0$$

Roots of this eqn.

$$D_1, D_2 = \frac{-2.5 \pm \sqrt{(2.5)^2 - 4}}{2}$$

$$= -0.5,\ -2$$

∴ The current eqn.

$$i(t) = K_1 e^{-0.5t} + K_2 e^{-2t} \quad \text{—(II)}$$

Initial condition

Assume, Before the s/w is closed, the circuit was deenergied

$\therefore\ i(0^-) = i(0^+) = 0 \qquad V_c(0^-) = V_c(0^+) = 0$

Applying initial condition to eqn.(II) & eqn.(I) at $t=0$

$5\times0 + 2\cdot di/dt + 2\times0 = 10$; $\boxed{\frac{di}{dt} = 5}$

$0 = K_1 + K_2$ —(III)

differenting eqn.(II)

$$\frac{di}{dt} = -0.5K_1e^{-0.5t} - 2K_2e^{-2t}$$

~~at t =~~ Applying initial condition at $t=0$

$5 = -0.5K_1 + 2K_2$ —(IV)

from (III) and (IV) (multiplying III by 0.5

0.5 × eq.III)

$$0.5\cancel{K_1} + 0.5K_2 = 0$$
$$-0.5\cancel{K_1} - 2K_2 = 5$$
$$-1.5K_2 = 5$$
$$\boxed{K_2 = -\frac{5}{1.5}} = -3.33$$

Putting value of K_2 in eqn.(III)

$$K_1 - \frac{5}{1.5} = 0$$
$$\boxed{K_1 = \frac{5}{1.5}} = 3.33$$

Putting K_1, K_2 in eqn.(II)

$$\boxed{i(t) = 3.33\left[e^{-0.5t} - e^{-2t}\right]}$$