MT-1 Solution

Semester IV, March 2020

Paper Code : ETEC 206, Paper : Network Analysis and Synthesis

Time : 1.5 Hours MM: 30

Note: Attempt Q.No 1 which is compulsory and any two more questions from the remaining.

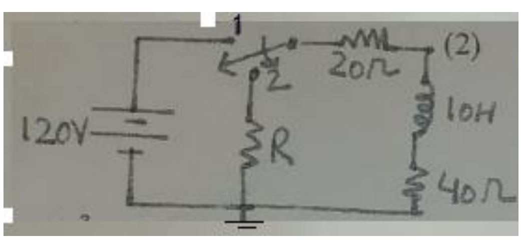
Q1(a) Check whether the given system is linear or not. (2)

Y[n] = x[n+1] – x[n-1]

Q1(b) Two ramp functions are given by f19t) = mtu(t) and f2 = m(t-a)u(t-a) where m1 and m2 are the slopes (+ve) and m1>m2. Draw final wave of these two functions

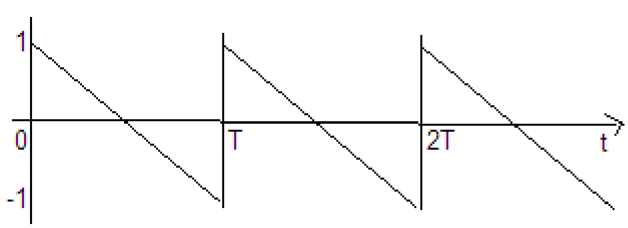
Q1(c) ramp step u(t-5) is applied to a series RL network. Calculate the current i(t), assume R=1ohm and L=1H.

Q1(d) A switch S has been in contact with point 1 for long time, it is moved to position 2 at t=0. If at t=0+, the voltage across the inductor is 120 V. Find the value of the resistance R.

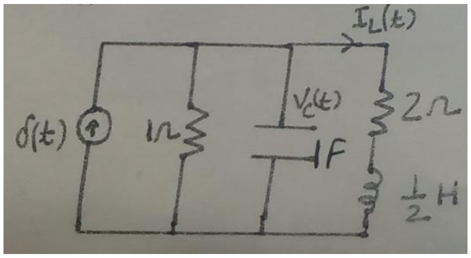


Q1(e) Find the value of F(0+), if F(s) = $\frac{5s^3-1600}{s(s^3+18s^2+90s+800)}$

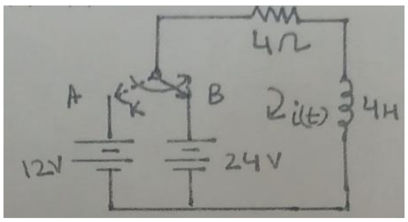
Q2(a) Determine the Laplace transform of the following periodic waveform. 5



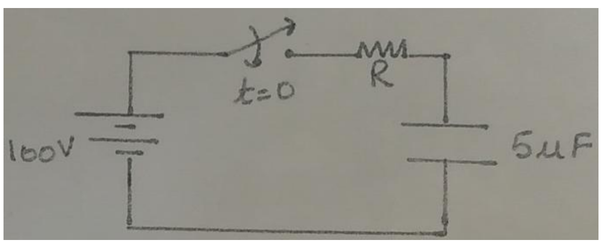
Q2(b) Find Vc(t) and IL(t) in the circuit below assuming zero initial condition. 5



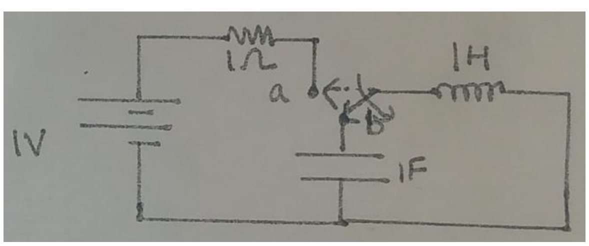
Q3(a) Find i(t) following switching of K at t=0min the circuit below from A to B 5



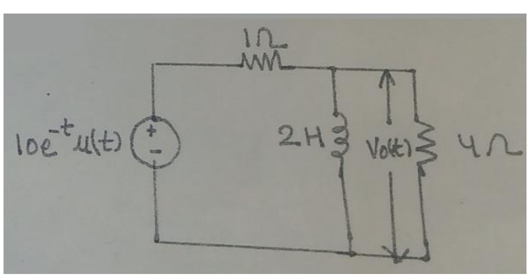
Q3(b) A resistance R and 5uF capacitor are connected in series across 100Vdc supply. Calculate the value of R such that the voltage across the capacitor below 50V in 5seconds after the circuit is switched on. 5



Q4(a) In the circuit shown below, steady state is reached with the switch in position “a”. At t=0, switching is done to “b” such that the circuit goes to discharging mode. Obtain the expression for the current i(t). 5



Q4(b) Assuming the initial current to be 2A through the inductor, find V0(t) in the figure below. 5



Solution: NAS MT-1 March 2020

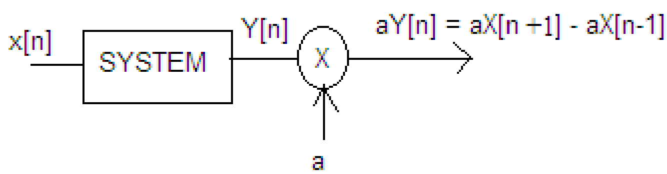
Q1(a) Check whether the given system is linear or not. **(2)**

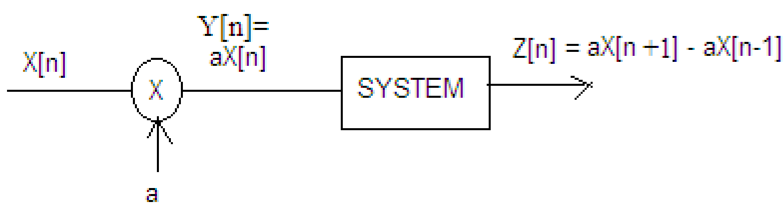
Y[n] = x[n+1] – x[n-1]

Solution:

We will check the system for the homogeneity and additivity

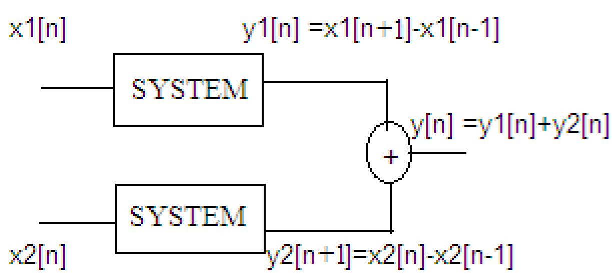
a. Checking for Homogeneity





Comparing Y[n] and Z[n] we find that the system is Homogeneous

b. Checking for additivity:

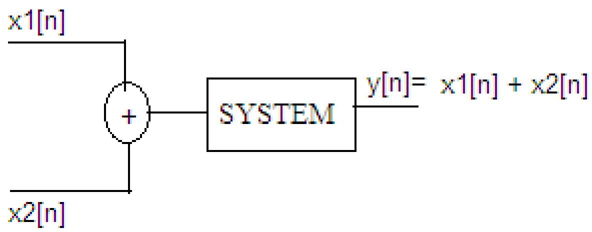


The response will be:

Y[n] = y1[n] + y2[n]

= x1[n+]-x1[n-1] + x2[n+]-x2[n-1] ------- (1)

Again:



Y[n] = x1[n] + x2[n]

= x1[n+1] – [x1[n-1] + x2[n+1] - x2[n-1]]

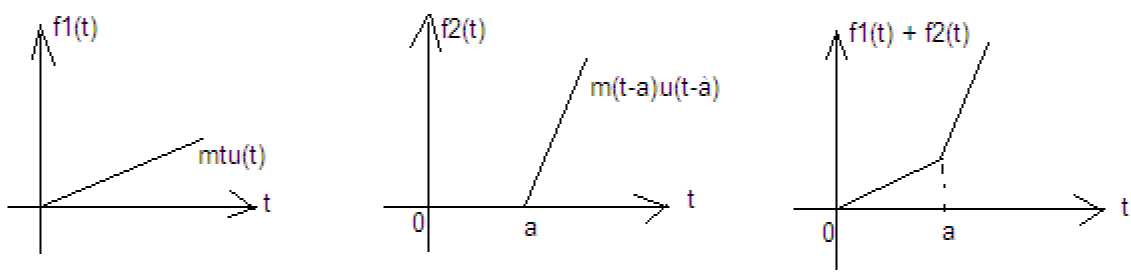
Rearranging

= x1[n]-x1[n-1] + x2[n]-x2[n-1] ------- (2)

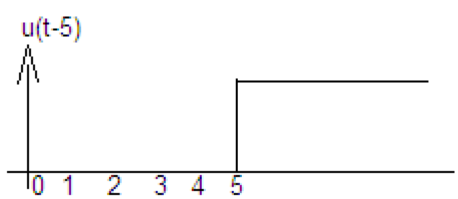
Comparing equation (1) and eq (2)

It is verified that the system satisfies both homogeneity and additivity we can say that the system is linear.

Q1(b) Two ramp functions are given by f1(t) = mtu(t) and f2 = m(t-a)u(t-a) where m1 and m2 are the slopes (+ve) and m1>m2. Draw final wave by adding of these two functions (2)



Q1(c) A unit step u(t-5) is applied to a series RL network. Calculate the current i(t), assume R=1ohm and L=1H. (2)



Input is shifted unit step

So, the initial current through the inductor:

$i(0^-) = i(0^+) = 0$

$Ri(t) + L di/dt = u(t-5)$

$RI(s) + LsI(s) – Li(0) = e^{-5s}/s$

$I(s)(s+1) = e^{-5s}/s$

$I(s) = e^{-5s}/s(s+1) = A/s + B/(s+1)$

$A = 1,\ B = -e^{5}$

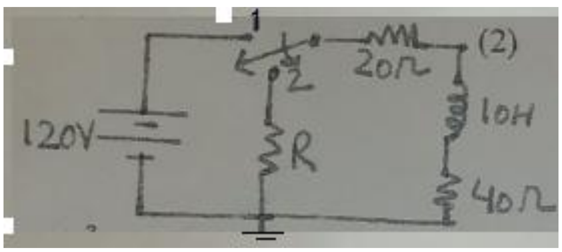
$I(s) = [1/s - e^{5}/(s+1)]$

Taking inverse Laplace we get

$i(t) \quad = [u(t) - e^{-(t-5)}\, u(t)]$

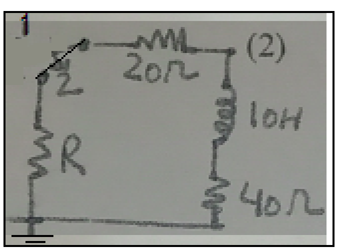
$\quad = (1 - e^{-(t-5)})u(t)$

Q1(d) A switch S has been in contact with point 1 for long time, it is moved to position 2 at t=0. If at t=0+, the voltage across the inductor is 120 V. Find the value of the resistance R. (2)



Solution: When the switch is at position 1 for a long time, the inductor acts as short so the final steady state current is $i(\infty) = V/R = 120/(20+40) = 120/60 = 2A$

When the switch is thrown to position 2,



$$L\frac{di}{dt} = (60 + R)i$$

$V_L = 2(60+R)$

$120 = 120 + 2R$

Therefore $\quad R=0$

Q1(e) Find the value of f(0+), if F(s) = $\frac{5s^3-1600}{s(s^3+18s^2+90s+800)}$ **(2)**

Solution Applying the final value theorem f(t) at

Lim(t→ 0^+) =Lim s→ ∞ [sF(s)]

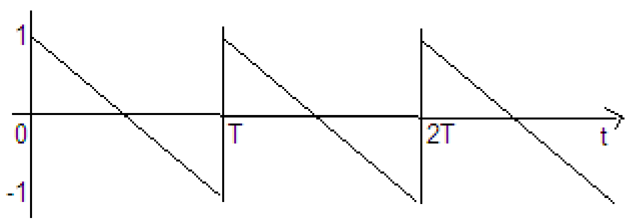
$$= \frac{\cancel{s}(5s^3-1600)}{\cancel{s}(s^3+18s^2+90s+800)}$$

$$Lim\ s \to \infty \frac{\cancel{s^3}(5-1600/s^3)}{\cancel{s^3}\left(1+\frac{18}{s}+\frac{90}{s^2}+800/s^3\right)}$$

$$= \frac{(5-0)}{(1+0+0+0)}$$

$f(0^+)$ = 5

Q2(a) Determine the Laplace transform of the following periodic waveform. 5



Solution Laplace of a periodic waveform is given as

$$f(t) = \frac{2}{T}\left(t-\frac{T}{2}\right)[u(t)-\ u(t-T)]$$

$$= -\frac{2}{T}\left(t-\frac{T}{2}\right)u(t) + \frac{2}{T}\left[(t-T)+\frac{T}{2}\right]u(t-T)$$

$$= \ -\frac{2}{T}\left[\frac{1}{s^2}-\frac{T}{2s}\right] + \left[\frac{T}{2s^2}\ +\frac{1}{s}\right]e^{-sT}$$

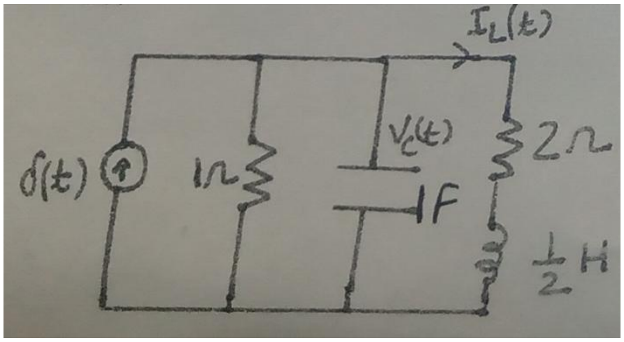
$$\frac{2}{Ts}\left[\frac{T}{2}(1+e^{-Ts}) - \frac{1}{s}(1-e^{-Ts})\right]$$

F(s) = £$f(t) = \ \frac{F(s)}{1-e^{-Ts}}$

$$= \frac{1}{1-e^{-Ts}}\frac{2}{Ts}\left[\frac{T}{2}\left(\frac{1+e^{-Ts}}{1-e^{-Ts}}\right)-\frac{1}{s}\right]$$

$$= \frac{2}{Ts}\left[\frac{T}{2}coth\left(\frac{Ts}{2}\right)-\frac{1}{s}\right]$$

Q2(b) Find Vc(t) and I_L(t) in the circuit below assuming zero initial condition. **5**



Applying the Nodal method

For zero intitial condition we can convert the circuit to s domain

$$\delta(t) = \frac{v_c}{R} + \frac{v_c}{Xc} + \frac{v_c}{Z_L}$$

Taking Laplace:

$$1 = \frac{Vc(s)}{1} + \frac{Vc(s)}{\frac{1}{s}} + \frac{V(s)}{2 + \frac{s}{2}}$$

$$1 = Vc(s)\left[1 + s + \frac{2}{s+4}\right]$$

$$1 = Vc(s)\left[\frac{s^2 + 5s + 6}{s+4}\right]$$

$$Vc(s) = \frac{s+4}{s^2 + 5s + 6}$$

$$Vc(s) = \frac{s+4}{(s+3)(s+2)} = \frac{A}{s+3} + \frac{B}{s+2}$$

Solving A= -1; B=2

$$\text{V(s)} = \frac{-1}{s+3} + \frac{2}{s+2}$$

Taking inverse Laplace

$$Vc(t) = -e^{-3t} + 2e^{-2t}$$

To find current through the inductor

$$I_L(s) = \frac{Vc(s)}{Z_L(s)} = \frac{\frac{s+4}{(s+3)(s+2)}}{\frac{s+4}{2}} = \frac{2}{(s+3)(s+2)} = \frac{A}{s+3} + \frac{B}{s+2}$$

Solving for A and B

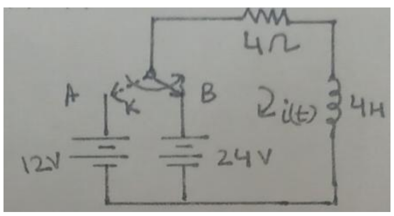
A=-2; B=2

$$I_L(s) = \frac{-2}{s+3} + \frac{2}{s+2}$$

Taking inverse Laplace

$$I_L(t) = 2(e^{-2t} - e^{-3t})Amp$$

Q3(a) Find i(t) following switching of K at t=0 in the circuit below from position A to B 5



Ans: Assuming the circuit has reached steady state before switching to B. At steady state iL(t) = V/R =12/4 = 3A

When the switch is at B

4di/dt + 4i=24

Taking Laplace

4sI(s) – Li(0-) + 4I(s) = 24/s

4sI(s) +4I(s) - 4x3 =24/s

4sI(s) +4I(s) =24/s+12

$$I(s) = \frac{12\left(\frac{2}{s}+1\right)}{4(s+1)}$$

$$I(s) = \frac{(6+3s)}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

Solving for A and B we get

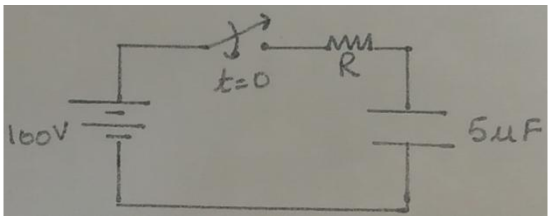
A=6; B= -3, putting these values back

$$I(s) = \frac{6}{s} - \frac{3}{s+1}$$

Applying inverse Laplace Transform we get

i(t) = 6u(t) - 3e^{-t}u(t)

Q3(b) A resistance R and 5uF capacitor are connected in series across 100Vdc supply. Calculate the value of R such that the voltage across the capacitor below 50V in 5seconds after the circuit is switched on. 5



Solution:

Assuming the circuit was initially relaxed, so, i(0-)=i(o+) = 0 and Vc(0-)=0V

At t>0, applying KVL

$$Ri + \frac{1}{C}\int i.dt = 100$$

Applying Laplace transform

$$RI(s) + \left[\frac{I(s)}{Cs} + \frac{v(0)}{s}\right] = \frac{100}{s}$$ Current direction

$$I(s)\left(R + \frac{1}{Cs}\right) + 0 = \frac{100}{s}$$

$$\frac{R}{s}I(s)\left(s + \frac{1}{RC}\right) = \frac{100}{s}$$

$$I(s) = \frac{100\cancel{s}}{R\cancel{s}(s + \frac{1}{RC})}$$

$$I(s) = \frac{100}{R(s + \frac{1}{RC})}$$

Applying inverse Laplace

$$i(t) = \frac{100}{R}e^{\frac{-t}{RC}}$$

$$V_c = \frac{1}{C}\int i.dt$$

$$V_c = \frac{1}{C}\int \frac{100}{R} e^{-t/RC} dt$$

$$V_c = \frac{100}{\cancel{RC}} \frac{e^{-t/RC}}{\frac{-1}{\cancel{RC}}} \qquad = 100e^{t/RC}$$

At t=5, Vc=50 therefore

$$50 = -100\, e^{\cancel{5}/R\cancel{5}x10^{-6}}$$

$$e^{\frac{10^6}{R}} = \frac{50}{100}$$

Taking log both sides

$$-\frac{10^6}{R} = \log\left(\frac{1}{2}\right)$$

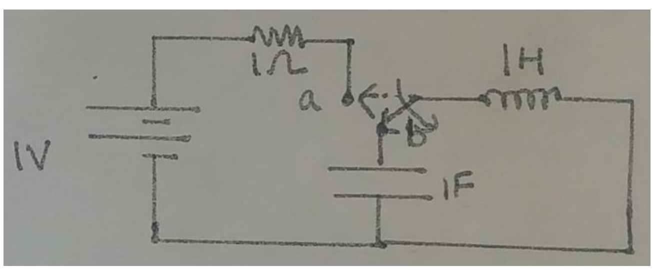
$$-\frac{10^6}{R} = -0.6931$$

$$R = 10^6/0.693$$

$$R = 10^6/0.3$$

R= 1.44M Ω

Q4(a) In the circuit shown below, steady state is reached with the switch in position "a". At t=0, switching is done to "b" such that the circuit goes to discharging mode. Obtain the expression for the current i(t). 5



Solution: At steady state, i(0-) =i(0+) = 1/1 = 1A

When the s/w is connected ay position 'b', applying KVL

$$L.\frac{di}{dt} + \frac{1}{C}\int idt = 0$$

Taking laplace transform

$$sLI(s) - Li(0-) + \frac{I(s)}{Cs} + \frac{Q}{s} = 0$$

$$sI(s) + \frac{I(s)}{Cs} - 1 + 0 = 0$$

$$I(s)(s + \frac{1}{s}) = 1$$

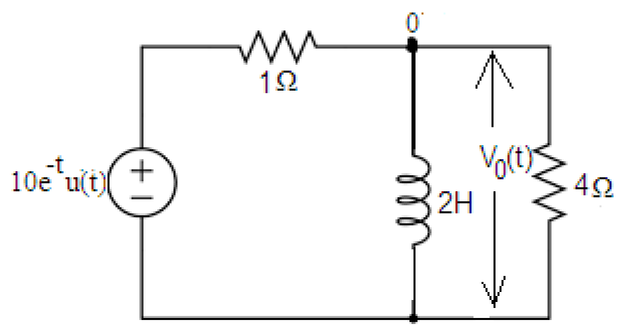
$$\frac{I(s)}{s}(s^2 + 1) = 1$$

$$I(s) = \frac{s}{s^2 + 1}$$

Applying the inverse Laplace transform we get

i(t) = cos(t)

Q4(b) Assuming the initial current to be 2A through the inductor, find V0(t) in the figure below. 5



Using the Node analysis at node ‘0’; the sum of all currents equals zero, therefore:

$$\frac{V_0 - 10e^{-t}u(t)}{R} + \frac{1}{L}\int V_0 dt + \frac{V_0}{4} = 0$$

$$V_0 - 10e^{-t}u(t) + \frac{1}{2}\int V_0 dt + \frac{V_0}{4} = 0$$

Taking the Laplace Transform

$$V_{0(s)} - \frac{10}{s+1} + \frac{V_0(s)}{2s} + \frac{i(0-)}{s} + \frac{V_0(s)}{4} = 0$$

$$V_{0(s)} - \frac{10}{s+1} + \frac{V_0(s)}{2s} + \frac{2}{s} + \frac{V_0(s)}{4} = 0$$

$$V_{0(s)}\left(1 + \frac{1}{2s} + \frac{1}{4}\right) = \frac{10}{s+1} - \frac{2}{s}$$

$$V_{0(s)}\left(\frac{5s+2}{4s}\right) = \frac{10s - 2(s+1)}{s(s+1)}$$

$$V_{0(s)5}\left(\frac{s+0.4}{4s}\right)=\frac{8s-2}{s(s+1)}$$

$$V_{0(s)}=\frac{4\cancel{s}(8s-2)}{5\cancel{s}(s+1)(s+0.4)}$$

$$V_{0(s)}=\frac{6.4s-1.6}{(s+1)(s+0.4)}=\frac{A}{s+1}+\frac{B}{s+0.4}$$

Solving we get A=13.33; B= 6.93

Substituting we get:

$$V_{0(s)}=\frac{13.33}{s+1}+\frac{6.93}{s+0.4}$$

Taking inverse Laplace transform we get:

$V_0(t) = 13.33e^{-t} + 6.93e^{-0.4t}$ volts