Open and Short Circuit Parameters

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Finding the Open Circuit Parameters or Z Parameters Z_{11}, Z_{12}, Z_{21}, Z_{22}

- We can find the network parameters by solving the network equations written using the loop or the nodal methods
- The Loop equations can be written as :

$V_1 = I_1 Z_{11} + I_2 Z_{12} + I_3 Z_{13} \ldots\ldots\ldots\ldots\ldots\ldots I_n Z_{1n}$

$V_2 = I_1 Z_{21} + I_2 Z_{22} + I_3 Z_{23} \ldots\ldots\ldots\ldots\ldots\ldots I_n Z_{2n}$

$V_3 = I_1 Z_{31} + I_2 Z_{32} + I_3 Z_{33} \ldots\ldots\ldots\ldots\ldots\ldots I_n Z_{3n}$

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$V_n = I_1 Z_{n1} + I_2 Z_{n2} + I_3 Z_{n3} \ldots\ldots\ldots\ldots\ldots\ldots I_n Z_{nn}$

Loop Equation

$V_1 = I_1 Z_{11} + I_2 Z_{12} + I_3 Z_{13} \ldots\ldots\ldots\ldots\ldots\ldots I_n Z_{1n}$
$V_2 = I_1 Z_{21} + I_2 Z_{22} + I_3 Z_{23} \ldots\ldots\ldots\ldots\ldots\ldots I_n Z_{2n}$
$V_3 = I_1 Z_{31} + I_2 Z_{32} + I_3 Z_{33} \ldots\ldots\ldots\ldots\ldots\ldots I_n Z_{3n}$
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$V_n = I_1 Z_{n1} + I_2 Z_{n2} + I_3 Z_{n3} \ldots\ldots\ldots\ldots\ldots\ldots I_n Z_{nn}$

In Matrix Form

$[V] = [Z][I]$

or $V = \Delta Z \times I$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ - \\ - \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13}\ldots & Z_{1n} \\ Z_{21} & Z_{22} & Z_{23}\ldots & Z_{2n} \\ Z_{31} & Z_{32} & Z_{33}\ldots & Z_{3n} \\ - & - & - & - \\ - & - & - & - \\ Z_{n1} & Z_{n2} & Z_{n3}\ldots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ - \\ - \\ I_n \end{bmatrix}$$

Finding Z

$$\Delta Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13}\ldots & Z_{1n} \\ Z_{21} & Z_{22} & Z_{23}\ldots & Z_{2n} \\ Z_{31} & Z_{32} & Z_{33}\ldots & Z_{3n} \\ - & - & - & - \\ - & - & - & - \\ Z_{n1} & Z_{n2} & Z_{n3}\ldots & Z_{nn} \end{bmatrix}$$

$$\Delta Z_{11} = \begin{bmatrix} Z_{22} & Z_{23}\ldots & Z_{2n} \\ Z_{32} & Z_{33}\ldots & Z_{3n} \\ - & - & - \\ - & - & - \\ Z_{n2} & Z_{n3}\ldots & Z_{nn} \end{bmatrix}$$

Then we can find the input impedance as: $Z_{in} = \Delta Z / \Delta Z_{11}$. We can similarly write all Z parameters like Z12, Z21, … Using Crammer's Rule

Two Port Network

- We can write the equations for the dependent variable in terms of the independent variables

  (V1,V2) = f(I1,I2)

  [V] =[Z][I]

- For a two-port network

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Z Parameter

- In order to determine the open circuit impedance or the Z parameter, open the input port and apply the excitation at the output port. Since the input is open I_1=0, we then determine the I_2 and V_1 to obtain Z_{12} and Z_{22}.

- When I_1 =0, the equation reduces to
  - $V_1=Z_{12}I_2$; so we get $Z_{12}= V_1/I_2$
  - $V_2=Z_{22}I_2$; so we get $Z_{22}= V_2/I_2$

- Then open the output port so that I2=0, and determine the I1 and V2 to obtain Z_{11} and Z_{21} . When I_2 =0, the equation reduces to
  - $V_1=Z_{11}I_1$; so we get $Z_{11}= V_1/I_1$
  - $V_2=Z_{21}I_1$; so we get $Z_{21}= V_2/I_1$

Equivalent Circuit of Loop equations

- We know the two loop equations:

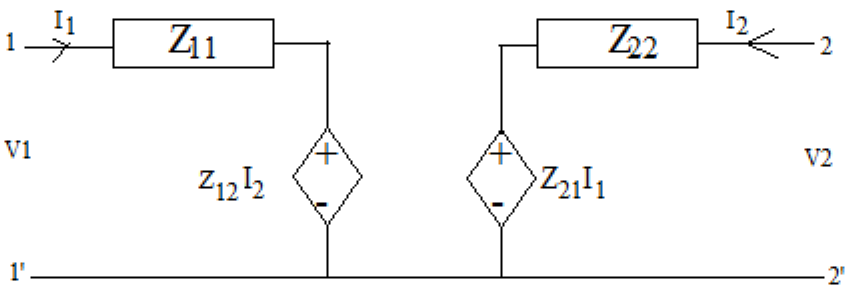
  $V_1 = Z_{11}I_1 + Z_{12}I_2$

  $V_2 = Z_{21}I_1 + Z_{22}I_2$

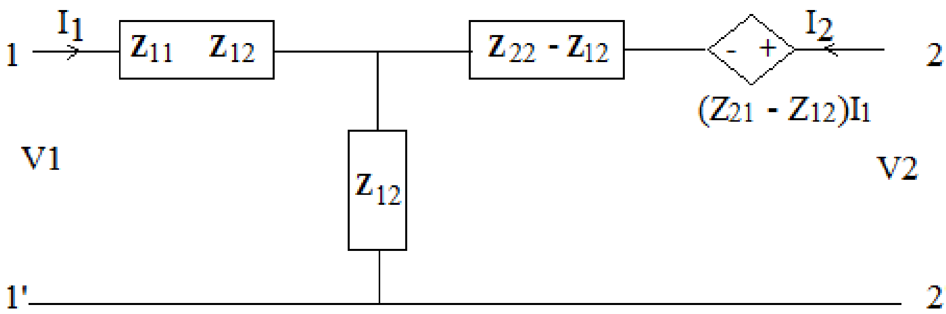
- Rewriting these equations remembering that : $Z_{12}I_2$ and $Z_{21}I_1$ are current controlled voltage sources (CCVS)

$V_1 = (Z_{11} - Z_{12})I_1 + Z_{12}(I_1 + I_2)$

$V_2 = (Z_{21} - Z_{12})I_1 + (Z_{22} - Z_{12})I_2 + Z_{12}(I_1 + I_2)$



(a) Equivalent Circuit of $V_1=(Z_{11} - Z_{12})I_1 + Z_{12}(I_1 + I_2)$



(b) Equivalent circuit of $V_2= (Z_{21}- Z_{12})I_1 + (Z_{22} - Z_{12})I_2 +Z_{12} (I_1 + I_2)$

Short Circuit Admittance (Y)

- The short circuit admittance is found by writing the node equations. These equations help us to write the dependent variables in terms of the independent variable as:
- (I1, I2) = f(V1, V2)

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

Which can be written in matrix form as:

[I] = [Y][V]

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Admittance Parameters

- First we short the output terminal, so V2=0 substituting we get:

$I_1=Y_{11}V_1$ or $Y_{11} = I_1 / V_1$

$I_2=Y_{21}V_1$ or $Y_{21} = I_2 / V_1$

- Then we short circuit the input port , so V1=0

$I_1= Y_{12}V_2$ or $Y_{12}= I_1 / V_2$

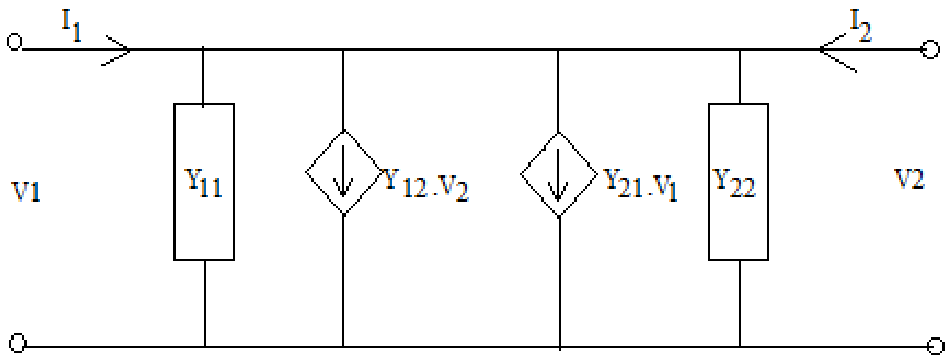
$I_2= Y_{22}V_2$ or $Y_{22} = I_2 / V_2$

Equivalent Circuit in terms of admittance

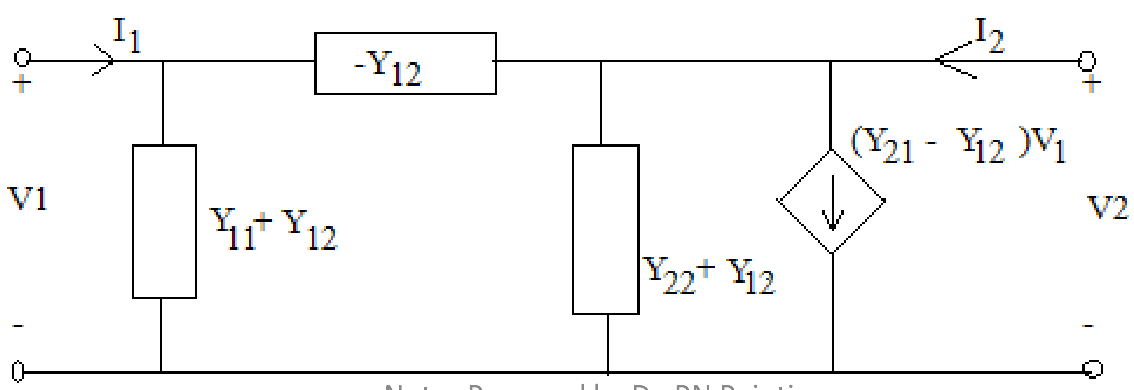
The equivalent circuit equations can be written by rearranging the above equation:

$I_1=(Y_{11}+ Y_{12})V_1 + Y_{12}(V_1 - V_2)$

$I_2=(Y_{21} - Y_{12})V_1 + (Y_{22} - Y_{12})V_2 + Y_{12}(V_2 - V_1)$



(a) Y Parameter Equivalent Circuit



Star-Delta Transformation

- The use of start-delta and delta-star sometimes helps in simplifying the network.
- **Delta-Star Transformation:**

Given Z1, Z2, Z3 then

1. Delta star Transformation
   - Let Z1, Z2, Z3 be impedance in delta then
   -

$$Z_a = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$$

$$Z_b = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3}$$

$$Z_c = \frac{Z_3 Z_1}{Z_1 + Z_2 + Z_3}$$

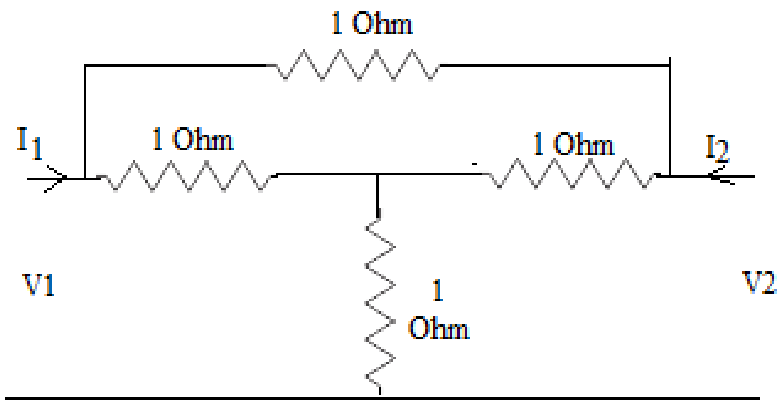
- **Star-Delta Transformation**
- Let Za, Zb, Zc be the impedances in star network then the impedance of delta network will be:

$$Z_1 = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_b}$$

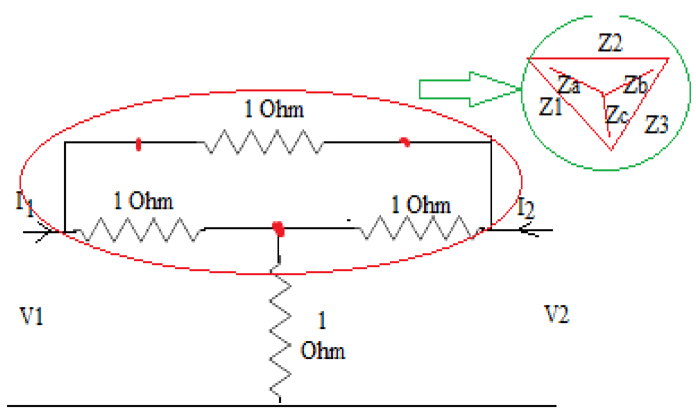
$$Z_2 = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c}$$

$$Z_3 = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_a}$$

Ex. Determine the Y parameter of the following network



- Ans:
- We know the Y-parameter equations :
- $I_1=Y_{11}V_1 + Y_{12}V_2$
- $I_2=Y_{21}V_1 + Y_{22}V_2$
- Let us first transform the delta connection at the top to star

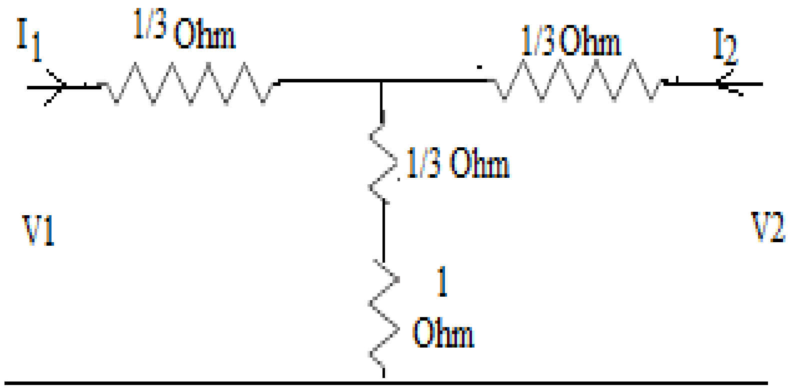


$$Z_a = \frac{1x1}{1+1+1} = \frac{1}{3}$$

$$Z_b = \frac{1x1}{1+1+1} = \frac{1}{3}$$

$$Z_c = \frac{1x1}{1+1+1} = \frac{1}{3}$$

So, the simplified network becomes:



Shorting the output port, V2=0

Therefore

V_1-1/3I_1 -4/3(I_1+I_2) =0 → V1 - 5/3I_1 -4/3I_2 = 0 (1)

and -1/3I_2 - 4/3(I_2-I_1) =0 → -5/3I_2 -4/3 I_1 =0

solving we get **I_2 = -4/5I_1** **and I_1 = -5/4I_2.** **(2)**

Substituting I2 from eqn (2) in eqn (1) we get

V_1 - 5/3I_1 - 4/3(-4/5I_1)= 0

Or V_1 = 9/15I_1 (3)

Therefore Y_{11} = I_1/V_1 = 15/9

Y_{11} = 5/3mho **(4)**

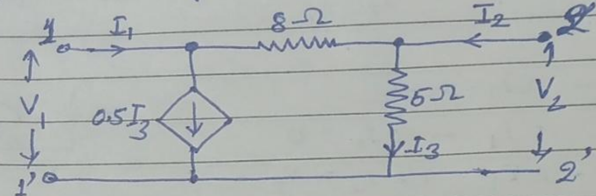
and also V_1 = 9/15x(-5/4I2)

So V_1= 3/4I_2 (Ignoring direction)

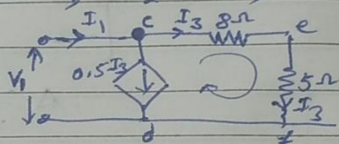
Y_{21} = I_2 **/** V_1 **= 4/3mho** **(5)**

- **(ii) Now short circuiting at the input port, V1=0**
- Applying KVL again
- V2 – 1/3I2 – 4/3(I2 + I1) =0 → V2 – 5/3I2 – 4/3 I1 =0 -(6)
- and - 1/3 I1 – 4/3 (I1 + I2) =0 → - 5/3I1 – 4/3I2 = 0 or
- I1= -4/5 I2 and I2 = -5/4I1
  (7)
- Substituting I1 in eqn (6) we get
- V2 – 5/3I2 – 4/3 (-4/5I2) =0 V_2= 9/15I_2
  (8)
- so, Y_{22} = I_2/V_2 15/9
- Or **Y_{22} = 5/3mho**
- From (7) and (8) V_2 = 9/15x(5/4I_1) =3/4I_1
- So V_2= 3/4I_1
- **Y_{12} = I_1/V_2 = 4/3mho**

Que A Π network has been shown in figure below, where $0.5 I_3$ is the controlled current source. Obtain the Z-parameters for this circuit



Sol. ⇒ With o/p open circuit at port 2-2', $I_2 = 0$,



Potential at node c-d = V.

$I_1 = I_3 + 0.5 I_3$

$= 1.5 I_3$

But $I_3 = \frac{V_{e-f}}{R} = \frac{V_1}{8+5} = \frac{V_1}{13}$

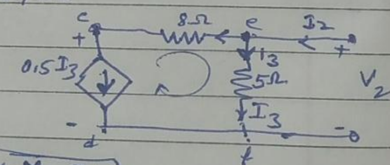
$\therefore \boxed{I_1 = \frac{1.5 \times V_1}{13} \text{ Amp}}$

Also $V_{ef} = V_2 = I_3 \times 5 = \frac{V_1}{13} \times 5 = \frac{5V_1}{13}$

$\therefore \boxed{V_2 = \frac{5V_1}{13}}$ $Z_{21} = \frac{V_2}{I_1} = \frac{5V_1 \times 13}{13 \times 1.5 V_1} = 3.33\,\Omega$

Therefore $Z_{11} = \frac{V_1}{I_1} = \frac{V_1}{\frac{1.5 V_1}{13}}$, OR $\boxed{Z_{11} = \frac{13}{1.5} = 8.67\,\Omega}$

⇒ Now open the input port so that $I_1 = 0$



As seen

$I_2 = I_3 + 0.5 I_3 =$

$\boxed{I_2 = 1.5 I_3}$

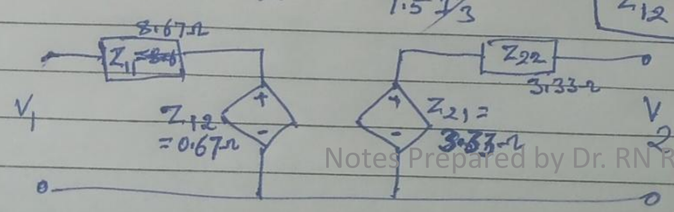
Loop-1 $5 I_3 - 8 \times 0.5 I_3 = V_{cd}$ but $V_{cd} \equiv V_1$ and $5 I_3 \equiv V_2$

$\therefore V_1 = 5 I_3 - 4 I_3$

$\boxed{V_1 = I_3}$

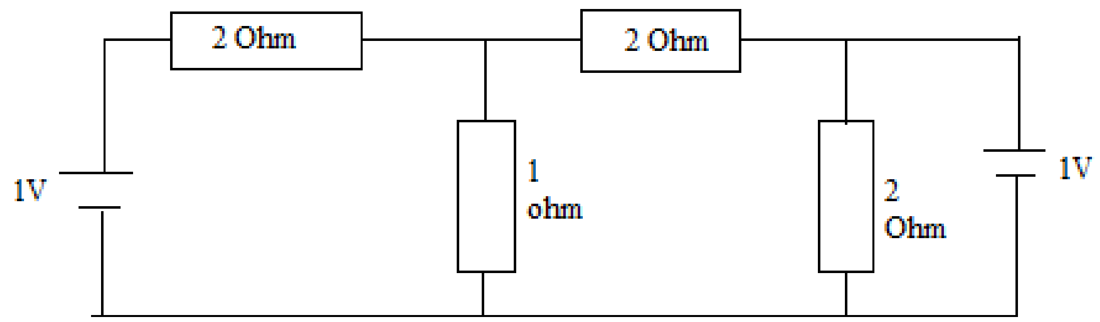
$\therefore Z_{22} = V_2 / I_2 = \frac{5 I_3}{1.5 I_3}$ $\boxed{Z_{22} = 3.33\,\Omega}$

$Z_{12} = V_1 / I_2 = \frac{I_3}{1.5 I_3}$ $\boxed{Z_{12} = 0.67\,\Omega}$



Questions

1. For the following network:
(a) Find the pen circuit impedance and draw the equivalent circuit
(b) Find the short circuit admittance and draw the equivalent circuit



2. For the following network, Determine the Z parameter