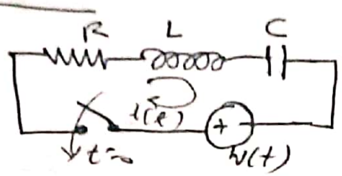
Series - RLC Circuit (S - Domain)

Assuming L, C are initially uncharged and s/w closed at $t=0$



$\therefore$ at $t=0$; $i(0^-) = i(0^+) = 0$
and $v_c(0^-) = v_c(0^+) = 0$

Applying KVL at $t>0$

$$R.i(t) + L.\frac{di}{dt} + \frac{1}{C}\int i\,dt = v(t)$$

Applying Laplace Transformation

$$R\,I(s) + sL\,I(s) - L\cancel{i(0)} + \frac{I(s)}{Cs} + \cancel{V(0)} = V(s)$$

$$I(s)\left[R + sL + \frac{1}{Cs}\right] = V(s) \quad \left(\begin{array}{c}\text{If } V \text{ is} \\ \text{constant}\end{array}\right)$$

$$I(s) = \frac{\cancel{s}\cancel{C}\,V/\cancel{s}}{L\cancel{C}\left[s^2 + \frac{R}{L}.s + \frac{1}{RC}\right]} = \frac{s.V/s}{LC\left(s^2 + \frac{R}{L}s + \frac{1}{RC}\right)}$$

$$= \frac{\cancel{s}\cancel{C}\,V/\cancel{s}}{L\cancel{C}\,(s-s_1)(s-s_2)}$$

where $s_1, s_2 = -\frac{R}{2L} \pm \sqrt{(R/2L)^2 - \left(\frac{1}{RC}\right)}$

$$I(s) = \frac{V}{L}\cdot\frac{1}{(s-s_1)(s-s_2)}$$

solving $= \frac{V}{L}\cdot\frac{1}{(s-s_1)(s-s_2)} = \frac{A}{s-s_1} + \frac{B}{s-s_2}$

$$A = \frac{V}{L}\Big/(s_1-s_2)$$

$$B = \frac{V}{L(s_2-s_1)}$$

Cont...

So, we get

$$I(s) = \frac{V}{L(s_1-s_2)}\left[\frac{1}{s-s_1} - \frac{1}{s-s_2}\right]$$

$s_1 \neq s_2$

$$i(t) = \frac{V}{L(s_1-s_2)}\left[e^{s_1t} - e^{s_2t}\right]$$

$s_1 = s_2$ Roots are same

$$I(s) = \frac{V/L}{(s-s_1)^2} \quad i$$

Taking inverse

$$i(t) = \frac{V}{L}\,t.e^{s_1t}$$

DAMPING.

(i) OVERDAMPED — In the Root If Roots are distinct

(1) overdamped $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

(ii) Critically Damped
If roots are −ve, Real, Repeatitive

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

(iii) Under Damped
Complex conjugate roots

$$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$

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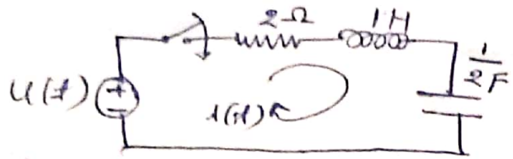
Q. Find $i(t)$ in the circuit shown in figure, if s/w is closed at $t = 0$



Sol.

Taking Laplace of circuit and Redrawing

$$I(s)\left[2+s+\frac{2}{s}\right] = \frac{1}{s}$$

$$I(s)\left[\frac{s^2+2s+2}{s}\right] = 1/s$$

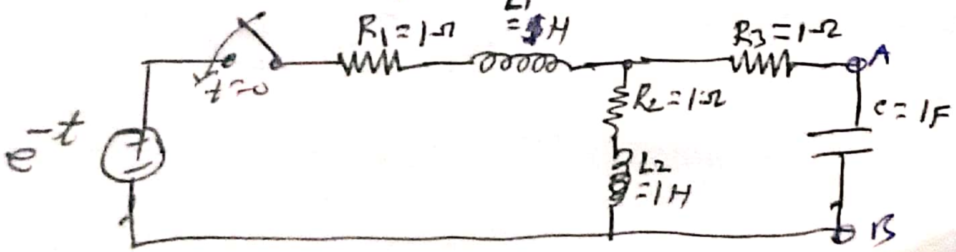
$$I(s) = \frac{1}{s} \times \frac{s}{s^2+2s+2}$$

$$= \frac{1}{(s+1)^2+(1)^2}$$

Taking Inverse laplace we get

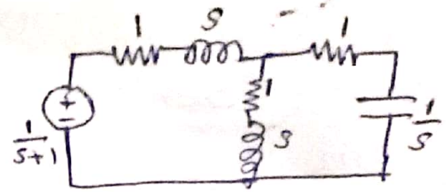
$$i(t) = e^{-t}.\sin(t)$$

Q. In a relaxed circuit, the s/w is closed at time $t=0$. Drawing Laplace transform network, and using thevenin theorem find $i(t)$ in resistor R_3

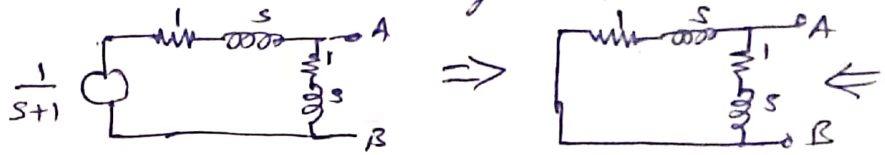


Sol.

(I) Drawing laplace of the circuit



(II) Applying thevenin theorem, (Remove Load and short voltage sources



Two impedance are in parallel

$$Z_{AB} = \frac{(1+s)(1+s)}{1+s+1+s} = \frac{(s+1)^2}{2(s+1)}$$

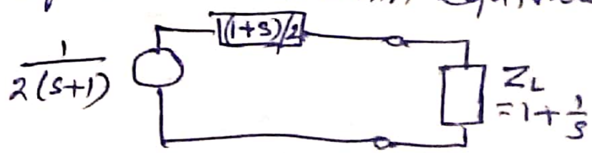
$$\boxed{Z_{AB} = \frac{s+1}{2}}$$

By voltage division $V_{AB} = V_{TH} = V \times \frac{Z_A}{Z_T}$

$$= \frac{1}{s+1} \times \frac{s+1}{2(s+1)}$$

$$V_{TH} = \frac{1}{2(s+1)}$$

Therefore the thevenin equivalent is:



$$I(s) = \frac{\frac{1}{2(s+1)}}{\frac{1+s}{2}+1+\frac{1}{s}} = \frac{s}{2(s+1)^2(s+2)}$$

$$= \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s+2}$$

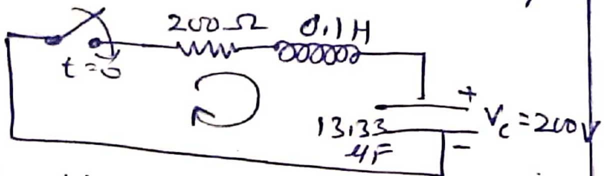
solving
A = 2
B = −1
C = −2

$$I(s) = \frac{2}{s+1} - \frac{1}{(s+1)^2} - \frac{2}{s+2}$$

$$\boxed{i(t) = 2e^{-t} - t.e^{-t} - 2e^{-2t}}$$

2015 end Term

For the circuit, obtain the current transient. Consider $V_c(0^-) = 200V$. If $C = 10\mu F$, is the solution critically damped



Sol.

$$R\,i(t) + L\cdot\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\cdot dt = 200$$

$$200\,I(t) + 0.1\frac{dI(t)}{dt} + \frac{1}{13.33\times10^{-6}}\int i(t)\,dt = 200$$

differentiating and Rearranging

$$\frac{d^2}{dt^2}i(t) + 2000\frac{d}{dt}i(t) + \frac{10^7}{13.33}\,i(t) = 0$$

Roots of this eqn. is

$$\alpha, \beta = 2000 \pm \sqrt{4000000 - \frac{4\times10^7}{13.33}}$$

$$= -500, \ -1500$$

$$\therefore i(t) = A\cdot e^{-500t} + B\cdot e^{-1500t}$$

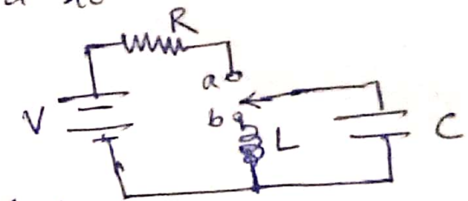
Condition for Critically Damped $D = 0$

$$B^2 - 4ac = 4000000 - 4\times1\times\frac{10}{C} = 0$$

$$\therefore C = 10\mu F$$

$\therefore$ Yes For $C = 10\mu F$, Solution is critically Damped.

Q.: Find the current $i(t)$ if the s/w is moved from a to b at time $t=0$



Sol.

When s/w is at 'a' for long steady state is reached and $V_c(0^-) = V$, also $i_L(0^-) = i_L(0^+) = 0$

At time $t=0$ s/w is moved to 'b'

$$\therefore L\cdot\frac{di}{dt} + \frac{1}{C}\int i\cdot dt = 0$$

Taking Laplace

$$sL\,I(s) - L\,i(0) + \frac{I(s)}{Cs} + \frac{V(0)}{s} = 0$$

Applying Initial condition

$$I(s)\left[Ls + \frac{1}{Cs}\right] = -\frac{V}{s}$$

$$\frac{L}{s}\,I(s)\left[s^2 + \frac{1}{LC}\right] = -\frac{V}{s}$$

$$I(s) = -\frac{V}{\cancel{s}}\times\frac{\cancel{s}}{L\left(s^2+\frac{1}{LC}\right)} = \frac{-V}{L\left(s^2+\left(\frac{1}{\sqrt{LC}}\right)^2\right)}$$

Taking Inverse Laplace Transform

$$I(s) = -\frac{V}{L}\times\frac{\sqrt{LC}}{\sqrt{LC}}\,\frac{1}{s^2+\left(\frac{1}{\sqrt{LC}}\right)^2}$$

$$i(t) = -V\sqrt{\frac{C}{L}}\cdot\sin\frac{1}{\sqrt{LC}}\,t$$