Resonance in Series and Parallel RLC Circuit

Resonance is defined as the state of the circuit when the current or voltage is maximum or minimum wrt the magnitude of the excitation at a particular frequency.

Series RLC Circuit The condition under the magnitude of the current is maximum, or the magnitude of the impedance is minimum, is called resonance.	**Parallel RLC Circuit:** The condition under which the magnitude of the total (supply) current is minimum, or the magnitude of the admittance is minimum (which means that the impedance is maximum), is called resonance.
Z= R + j(X_L-X_C) $At\ resonance\quad X_L = X_C$ Therefore the current at resonance is I = V/R Also $j\omega L = \frac{1}{j\omega C}$ $\omega = \sqrt{\frac{1}{LC}}$ $frequency\ f_r = \frac{1}{2\pi\sqrt{LC}}$ or $\boldsymbol{\omega_r = \frac{1}{\sqrt{LC}}}$	$Y = \frac{1}{R} + \frac{1}{X_L} + \frac{1}{X_C}$ $Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$ $= \frac{1}{R} + \frac{1}{2\pi f L} + 2\pi f C$ At resonance $X_L = X_C$ $2\pi f L = \frac{1}{2\pi f C}$ Or $f_r = \frac{1}{2\pi\sqrt{LC}}$ Or $\boldsymbol{\omega_r = \frac{1}{\sqrt{LC}}}$
At resonance Current $I = \frac{V}{Z} = \frac{V}{R + j(X_L - X_C)}$ As Xl = XC at resonance, therefore $I = \frac{V}{Z} = \frac{V}{R}$	In parallel circuit total current is I = IR + IXL + IXC Where IR = V/R; IC = V/XC = ; IL = V/XL $I_R = {}^{V}/_{R}\ ;\ I_L = {}^{V}/_{X_L} = {}^{V}/_{2\pi f L}\ ;\ I_C = V2\pi f C$ Total current $I = I_R + I_L + I_C$ Vector sum = $I_T = \sqrt{IR^2 + (I_L + I_C)^2}$ But at resonance $I_L equal\ I_C\ cancelling\ giving\ net\ reactaive\ curren$ Therefore $I = I_R = \frac{V}{R}$

Power factor of the circuit at resonance becomes :

$$\cos\emptyset = \frac{R}{Z} = \frac{R}{R} = 1$$

Properties of resonance of Series RLC Circuit	
1. Power factor of the circuit is unity	1. Power factor is unity

2. Voltage and current are in phase 3. Net reactance is zero since XL = XC or XL - XC = 0. 4. Circuit has minimum impedance and maximum admittance 5. Current in the circuit is maximum I = V/R 6. the use of either pure or impure components in the series RLC circuit does not affect the calculation of the resonance frequency, $f_r = \frac{1}{2\pi\sqrt{LC}}$	2. total circuit current is "in-phase" with the supply voltage as the two reactive components cancels each other 3. At resonance the admittance of the circuit is minimum and equals the conductance of the circuit 4. the use of either pure or impure components in the parallel RLC circuit affects the calculation of the resonance frequency, and is for Pure inductor: $f_r = \frac{1}{2\pi\sqrt{LC}}$ for impure inductor $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{Rs^2}{L^2}}$

Q Factor of series and Parallel Resonating circuit

Q Factor of series Resonating circuit	**Q Factor of Parallel Resonating circuit**
Q factor in a series RLC circuit may be defined as the voltage magnification in the circuit at resonance. It is the ratio of the voltage across inductor or capacitor to the applied input voltage. $Q = \frac{V_L}{V} = \frac{V_C}{V}$ $Q = \frac{I.X_L}{I.R} = \frac{X_L}{R} = \frac{\omega_0 L}{R}$ or $Q = \frac{\omega_0 L}{R} = \frac{1}{\sqrt{LC}}\frac{L}{R}$ $= \frac{1}{R}\sqrt{\frac{L}{C}}$ $also\ Q = \frac{I.X_C}{I.R} = \frac{X_C}{R} = \frac{1}{\omega_0 CR}$ Or $Q = \frac{1}{\omega_0 CR} = \frac{1}{(1/\sqrt{LC})RC}$ $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$	The selectivity or Q-factor for a parallel resonance circuit is generally defined as the ratio of the circulating branch currents to the supply current and is given as $Q = \frac{R}{X_L} = \frac{R}{X_C}$ Or $Q = \frac{R}{2\pi f L} = 2\pi f CR$ $= R\sqrt{\frac{C}{L}}$ Also $Q = 2\pi f CR = \omega_0 CR$ $= \frac{1}{\sqrt{LC}} CR$ $= R\sqrt{\frac{C}{L}}$
Note that the Q-factor of a parallel resonance circuit is the inverse of the expression	

for the Q-factor of the series circuit. Also in series resonance circuits the Q-factor gives the voltage magnification of the circuit, whereas in a parallel circuit it gives the current magnification.

Voltage across the Inductor

$$V_L = I.X_L = \frac{V}{R}X_L = \frac{\omega_0 LV}{R}$$

or

$$V_L = Q.V\ volts$$

Voltage across the Capacitor

$$V_C = I.X_C = \frac{V}{R}X_C = \frac{V}{\omega_0 RC}$$

or

$$V_C = Q.V\ volts$$

Bandwidth

The bandwidth is the difference between the half power frequencies Bandwidth = B $=\omega 2 - \omega 1$

Bandwidth in Series RLC circuit	Bandwidth in Parallel RLC circuit
B $=\omega 2 - \omega 1$ or $= R/L$	B $=\omega 2 - \omega 1$ or $B = 1/RC$

Half Power Frequency in Series and Parallel RLC circuit

Let

f_0 be the resonance frequency and

f_1 be the frequency when the net circuit reactance is –ve

f_2 be the frequency when the net circuit reactance is +ve

The band within lower and upper half power frequency is called the bandwidth of the resonating circuit.

Also we know that

$$\omega_2\omega_1 = \frac{1}{LC}$$

And

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Comparing above two equations

$$\omega_0^2 = \omega_2\omega_1 \qquad \omega_0 = \sqrt{\omega_2\omega_1}$$

Half Power Frequency in series RLC circuit	Half Power Frequency in Parallel RLC circuit
$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{\omega 0^2}}$ $\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{\omega 0^2}}$ Where $\omega_0 = \frac{1}{\sqrt{LC}}$	$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{\omega 0^2}}$ $\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{\omega 0^2}}$ $where\ \omega_0 = \frac{1}{\sqrt{LC}}$

Frequency at which voltage across the inductor and capacitor is maximum

We can also calculate the frequencies at which voltage V_C across the capacitor is maximum:

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

We can also calculate the frequencies at which voltage V_L across the inductor is maximum:

$$f_r = \frac{1}{2\pi\sqrt{LC - \frac{c^2R^2}{2}}}$$

Solved Questions

1. Calculate the half power frequencies of a series resonant circuit where the resonant frequency is 150x 10^3 Hz and the bandwidth is 75KHz

Solution:

$f_2 - f_1$ = bandwidth

$f_2 - f_1 = 75$ --------------1

and

$\omega_r = \sqrt{\omega_2 \omega_1}$

or

$\sqrt{f_2 f_1} = f_r = 150$ --------------2

Solving eq 1 and eq 2 we get

$f_1 = 117$kHz

$f_2 = 192$kHz

Que: 2

2016 end term

Derive the Q of the Series/parallel circuit given with Vs = V_mSin(ωt)

Ans:

Total Energy Stored in the circuit=

V = Vm sin ωt

IL = Vm Sin $(\omega t - 90)$

$$I_L = \frac{V_m Sin(\omega t - 90)}{X_L}$$

$$= \frac{V_m}{\omega_0 L} \cos(\omega_0 t)$$

Total Energy Stored $= \frac{1}{2} LI^2 + \frac{1}{2} CV^2$

$$= \frac{1}{2} \frac{LVm^2 cos^2(\omega_0 t)}{\omega 0^2 L^2} + \frac{1}{2} CVm^2 Sin(\omega_0 t)$$

As $cos^2(\omega_0 t) + sin^2(\omega_0 t) = 1$

Therefore, Total Energy Stored $= \frac{1}{2} CVm^2$

The average power dissipated in a resonant circuit can be expressed in terms of the **rms** voltage and current as follows:

$$P_{avg} = I^2{}_{rms}R = \frac{V^2{}_{rms}}{Z^2}R = \frac{V^2{}_{rms}R}{R^2 + (X_L - X_C)^2}$$

Now, $(X_L - X_C)^2 = \left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{L^2}{\omega^2}(\omega^2 - \omega^2{}_0)^2$

Where, the resonant frequency expression

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Substitution now gives the expression for average power as a function of frequency.

$$P_{avg} = \frac{V^2{}_{rms}R\omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega^2{}_0)^2}$$

The average power at resonance is just:

$$P_{avg} = \frac{V^2{}_{rms}}{R} = \left(\frac{Vm}{\sqrt{2}}\right)^2$$

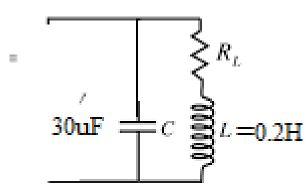
Energy Loss Per Cycle $= \frac{2\pi}{\omega_0}(Avg\ Power)$

$$Quality\ Factor = 2\pi \frac{Maximum\ energy\ stored\ in\ the\ circuit}{Total\ Energy\ Loss\ Per\ Cycle}$$

$$= \frac{2\pi\left(\frac{1}{2}CVm^2\right)}{\pi\left((V^2{}_m/\omega_0\right)} = \omega_0 CR = \frac{R}{\omega_0 L}$$

Question-3: End Term 2016

Compare the resonant frequency of the circuit shown in figure for R=0 ohm to that R= 50Ohm



Solution:

$$Y = \frac{1}{X_C} + \frac{1}{X_L}$$

$$= j\omega C + \frac{1}{R + j\omega L}$$

$$= j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

$$= \frac{R}{R^2 + \omega^2 L^2} + j\omega\left(C - \frac{L}{R^2 + \omega^2 L^2}\right)$$

At resonance imaginary part=0, therefre

$$C - \frac{L}{R^2 + \omega^2 L^2} = 0$$

Solving, we get

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Now as per question:

a. At R = 0;

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{0.2X30X10^{-6}}}$$

b. At R=50 ohm

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{0.2X30X10^{-6}} - \frac{50^2}{0.1^2}}$$